

**CH2 - INVERSE TRIGNOMETRIC****❖ Very Short (Objective Type) / Short Answer Type**

Q1. If  $\sin^{-1}x = \pi/10$ , for some  $x \in \mathbb{R}$ , then the value of  $\cos^{-1}x$  is \_\_\_\_.

Q2. If  $\tan^{-1}x - \cot^{-1}x = \pi/6$ , then  $x$  is \_\_\_\_.

Q3. The domain of the function  $y = \sin^{-1}(-x^2)$  is

- (a)  $[0, 1]$                       (b)  $(0, 1)$   
(c)  $[-1, 1]$                       (d)  $\varnothing$

Q4. If  $\sec^{-1}x + \sec^{-1}y = \pi/2$ , the value of  $\operatorname{cosec}^{-1}x + \operatorname{cosec}^{-1}y$  is

- (a)  $\pi$                               (b)  $\pi/2$   
(c)  $3\pi/2$                           (d)  $\geq -\pi$

Q5. The value of  $\tan^2(\sec^{-1}2) + \cot^2(\operatorname{cosec}^{-1}3)$  is

- (a) 5                                  (b) 11  
(c) 13                                (d) 15

Q6.  $\cot(\pi/4 - 2 \cot^{-1}3) = 7$ , state true or false.

Q7. If  $3 \sin^{-1}x + \cos^{-1}x = \pi$ , then  $x$  is equal to

- (a) 0                                  (b)  $1/\sqrt{2}$   
(c) -1                                (d)  $1/2$

Q8. Show that

$$\sin^{-1}(2x\sqrt{1-x^2}) = 2 \sin^{-1}x, -1/\sqrt{2} \leq x \leq 1/\sqrt{2}$$

[NCERT]

Q9. Find the value of  $\sec(\tan^{-1}2)$ .

[NCERT Exemplar]

Q10. If  $\sin(\sin^{-1}1/5 + \cos^{-1}x) = 1$ , then find the value of  $x$ .

[NCERT; HOTS; Delhi 2014]

Q11. Find the value of  $\tan^{-1}(x/y) - \tan^{-1}((x-y)/(x+y))$ .

[NCERT; Delhi 2013 (C), 2011]

# LOYAL EDUCATION MATHEMATICS

Result Oriented

( DAILY PRACTICE PAPER )

[ CLASS XII ]

Q12. If  $y = \cot^{-1}(\sqrt{\cos x}) - \tan^{-1}(\sqrt{\cos x})$ , then prove that  $\sin y = \tan^2(x/2)$ .

[Foreign 2013]

Q13. Prove that  $3 \sin^{-1}x = \sin^{-1}(3x-4x^3)$ ,  $x \in [-1/2, 1/2]$ .

[CBSE 2018]

Q14. Prove that  $\sin^{-1}x = \tan^{-1}(x/\sqrt{1-x^2})$ .

Q15. Write  $\cot^{-1}(1/\sqrt{x^2-1})$ ,  $|x| > 1$  in the simplest form.

[Foreign 2013]

Q16. If  $\sin^{-1}x + \sin^{-1}y = 2\pi/3$ , then find the value of  $\cos^{-1}x + \cos^{-1}y$ .

Q17. Find the value of  $\tan(2 \tan^{-1}(1/5))$ .

[Delhi 2013]

## ❖ Long Answer I / Long Answer II Type

Q18. Prove that

$$\tan(\pi/4 + 1/2 \cos^{-1}(a/b)) + \tan(\pi/4 - 1/2 \cos^{-1}(a/b)) = 2b/a$$

[DoE; Delhi 2017]

Q19. Prove that  $\sin^{-1}(4/5) - \sin^{-1}(5/13) - \sin^{-1}(16/65) = \pi/2$

Q20. Prove the following:  $\tan^{-1}\sqrt{x} = 1/2 \cos^{-1}((1-x)/(1+x))$   $x \in [0, 1]$

[NCERT]

Q21. Prove that  $\tan^{-1}((\sqrt{1+x}-\sqrt{1-x})/(\sqrt{1+x}+\sqrt{1-x})) = \pi/4 - 1/2 \cos^{-1}x$ ,  $-1/\sqrt{2} \leq x \leq 1$

[NCERT; DoE: AI 2011]

Q22. Prove the following:

$$\cos[\tan^{-1}(\sin(\cot^{-1}x))] = \sqrt{(1+x^2)}/\sqrt{(2+x^2)}$$

Q23. Prove the following:

$$\cos(\sin^{-1}(3/5) + \cot^{-1}(3/2)) = 6/(5\sqrt{13})$$

Q24. If  $x = a \cos \theta + b \sin \theta$ ,  $y = a \sin \theta - b \cos \theta$ , prove that  $x^2/a^2 + y^2/b^2 = 1$ .

[MOTO, 2008]

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[ CLASS XII ]

Q25. Show that  $\tan (\frac{1}{2} \sin^{-1} 3/5) = 4 - \sqrt{7}/3$ . [IIT 2003]

Q26. Prove That  $\sec^2 (\tan^{-1} 2) - \operatorname{cosec}^2 (\cot^{-1} 3) = 5$ .

Q27. If  $a_1, a_2, a_3, \dots, a_n$  is an arithmetic progression with common difference  $d$ , then evaluate the following:

$$\tan \left[ \tan^{-1} \left( \frac{d}{1+a_1 a_2} \right) - \tan^{-1} \left( \frac{d}{1+a_2 a_3} \right) - \tan^{-1} \left( \frac{d}{1+a_3 a_4} \right) - \tan^{-1} \left( \frac{d}{1+a_{n-1} a_n} \right) \right]$$

Q28. Prove that  $\tan^{-1} (1/5) - \sec^{-1} (\sqrt{(5\sqrt{2})}/7) - \tan^{-1}(1/8) = \pi / 4$

Q29. Write the following function in the simplest form:

$$\tan^{-1} (\cos x / (1 + \sin x))$$

Q30. Simplify:

$$\tan^{-1} ((a \cos x - b \sin x) / (b \cos x + a \sin x)), \text{ if } a/b \tan x > -1.$$

[NCERT]

Q31. Write the following function in the simplest form:

Q32. Solve for  $x$ ,  $\tan^{-1} ((1-x)/(1+x)) = 1/2 \tan^{-1}(x)$ ,  $x > 0$ .

[NCERT; NCERT Exemplar; Foreign 2011]

Q33. Solve the following equation:

$$\tan^{-1} ((x+1)/(x-1)) + \tan^{-1} ((x-1)/(x-1)) = \tan^{-1} (-7).$$

Q34. Write the following function in the simplest form:

$$\tan^{-1}(\cos x - \sin x)/(\cos x + \sin x), 0 < x < \pi$$

[NCERT]

Q35. If  $\tan^{-1}(x-1)/(x-2) + \tan^{-1}(x+1)/(x+2) = \pi/4$ , then find the value of  $x$ .

[NCERT; Foreign 2013]

Q36. Solve the following equation:

$$\cos (\tan^{-1} x) = \sin (\cot^{-1} 3/4)$$

[NCERT Exemplar; Delhi 2017; Foreign 2014; A1 2013]

Q37. Find the value of the expression

$$\sin (2 \tan^{-1} 1/5) - \cos (\tan^{-1} 2\sqrt{2}).$$

# LOYAL EDUCATION MATHEMATICS

Result Oriented

( DAILY PRACTICE PAPER )

[ CLASS XII ]

Q38. Find the value of  $\cot \frac{1}{2} [\cos^{-1} \frac{2x}{1+x^2} + \sin^{-1} \frac{1-y^2}{1+y^2}]$   
 $|x| < 1, y > 0$  and  $xy < 1$ .

[Foreign 2017]

Q39. Prove that:  $\tan^{-1}(\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}})$

Q40.  $\cos^{-1}(\cos 13\pi/6)$ .

Q41. Write the principal value of the following:  $\sin^{-1}(\sin 4\pi/5)$ .

Q42. Using principal value evaluate the following:  
 $\cos^{-1}(\cos 2\pi/3) + \sin^{-1}(\sin 2\pi/3)$ . [AI 2011]

Q43. If  $\sin^{-1}x + \sin^{-1}y = \pi$ , then find the values of  $x$  and  $y$ .

Q44. If  $x < y < 0$ , such that  $xy = 1$ , then find the value of  $\tan^{-1}x + \tan^{-1}y$ .

Prove that in Question 45 to 55:

Q45.  $\sin^{-1}(12/13) + \cos^{-1}(4/5) + \tan^{-1}(63/16) = \pi$ . [NCERT]

Q46.  $\cot^{-1}\sqrt{\frac{(1 + \sin x) + \sqrt{1 - \sin x}}{(1 + \sin x) - \sqrt{1 - \sin x}}} = x/2$ ,  
 $x \in (0, \pi/4)$  [NCERT; Delhi 2014, 11]

Q47.  $\tan^{-1}(1/4) + \tan^{-1}(2/9) = 1/2 \cos^{-1}(3/5)$  [HOTS]

Q48.  $\cos^{-1}(12/13) + \sin^{-1}(3/5) = \sin^{-1}(56/65)$   
[NCERT; Dehradun 2019]

Q49.  $\tan^{-1}2x + \tan^{-1}(4x/(1-4x^2)) = \tan^{-1}((6x-8x^3)/(1-12x^2))$ ,  
 $|x| < 1/(2\sqrt{3})$  [Foreign 2017]

Q50.  $\tan^{-1}(1/2) + \tan^{-1}(1/7) = \tan^{-1}(31/17)$  [NCERT; AI 2011]

Q51.  $\sin^{-1}(8/17) + \sin^{-1}(3/5) = \cos^{-1}(36/85) = \tan^{-1}(77/36)$   
[NCERT; Delhi 2012, 13 (C)]

Q52.  $\sin^{-1}(1/\sqrt{5}) + \cot^{-1}(3) = \pi/4$ . [HOTS]

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[ CLASS XII ]

Q53.  $\cos(\sin^{-1}(3/5) + \sin^{-1}(5/13)) = 33/65.$

Q54.  $\cos^{-1}(4/5) + \cos^{-1}(12/13) = \cos^{-1}(33/65).$  [NCERT]

Q55.  $2 \sin^{-1}(3/5) = \tan^{-1}(24/7).$

Write the following functions in the simplest form (Question 56 and 27):

Q56.  $\tan^{-1}(3x - x^3)/(1 - 3x^2)$

Q57.  $\tan^{-1}\sqrt{(1 - \cos 3x)/(1 + \cos 3x)}, x < \pi$  [NCERT]

Q58. Solve for x,  $\tan^{-1}(x + 1) + \tan^{-1}(x - 1) = \tan^{-1}(8/31)$   
[Foreign 2015]

Q59. Find the value of x, if  $\sin [\cot^{-1}(x + 1)] = \cos (\tan^{-1}x)$   
[DoE; Bhubaneswar 2015, Delhi 2015]

Q60. Prove the following:  $2 \sin^{-1}(3/5) - \tan^{-1}(17/31) = \pi/4$   
[Bhubaneswar 2015]

Q61. Solve the following for x:  
 $\tan^{-1}((x - 2)/(x - 3)) + \tan^{-1}((x + 2)/(x + 3)) = \pi/4, |x| < 1$   
[Patna 2015]

Q62. Prove the following:  $\sin [\tan^{-1}((1 - x^2)/2x)] + \cos^{-1}((1 - x^2)/(1 + x^2)) = 1, 0 < x < 1$   
[Guwahati 2015]

Q63. Solve for x,  $2 \tan^{-1}(\sin x) = \tan^{-1}(2 \sec x), x \neq \pi/2$   
[DoE; Foreign 2012]

Q64. Solve for x:  $\tan^{-1}(2x/(1-x^2)) + \cot^{-1}((1-x^2)/2x) = 2\pi/3, x > 0.$

Q65. Solve for x:  $\cos^{-1}((x^2 - 1)/(x^2 + 1)) + 1/2 \tan^{-1}(2x/(1-x^2)) = 2\pi/3$

Q66. Solve for x:  $\sin^{-1}(2\alpha/(1 + \alpha^2)) + \sin^{-1}(2\beta/(1 + \beta^2)) = 2 \tan^{-1}x.$   
[HOTS]

Q67. Solve for x:  $\sin^{-1}x + \sin^{-1}2x = \pi/3$  [HOTS]

❖ **INTEGRATED EXERCISE**

**Very Short (Objective Type) / Short Answer Type**

1. The principal value of  $\sin^{-1}(\sin 2\pi/3)$  is  
(a)  $2\pi/3$  (b)  $\pi/3$  (c)  $-\pi/6$  (d)  $\pi/6$
2. The value of  $\cos^{-1}(1/2) + 3 \sin^{-1}(1/2)$  is equal to  
(a)  $\pi/4$  (b)  $\pi/6$  (c)  $2\pi/3$  (d)  $5\pi/6$
3. The greatest and least values of  $(\sin^{-1}x)^2 + (\cos^{-1}x)^2$  are respectively \_\_\_\_\_.
4. The value of  $\sin(2 \sin^{-1}(-6))$  is \_\_\_\_\_.
5. Find the principal value of  $\operatorname{cosec}^{-1}(2)$ . [NCERT]
6. Evaluate  $\tan^{-1}(\sin(-\pi/2))$ . [NCERT Exemplar]
7. Write the value of  $\cos^{-1}(-1/2) + 2 \sin^{-1}(1/2)$ .
8. Write one branch of  $\sin^{-1}x$  other than the principal branch.
9. Find the principal value of  $\tan^{-1}(-1)$
10. Find the principal value of  $\cos^{-1}(\cos(7\pi/6))$
11. Find the value of  $\sin(2 \sin^{-1}(3/5))$ .
12. Find the principal value of  $\tan^{-1}(\tan(9\pi/8))$
13. Write the principal value of  $\tan^{-1}(\tan(3\pi/4))$ .

**❖ Long Answer I / Long Answer II Type**

14. Prove that  $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \pi$

15. Prove the following:  $(9\pi/8) - (9/4) \sin^{-1}(1/3) = (9/4) - \sin^{-1}(2\sqrt{2}/3)$

16. Show that

$$2 \tan^{-1} \{ \tan(\alpha/2) \tan(\pi/4 - \beta/2) \} = \tan^{-1}((\sin \alpha \cos \beta)/(\cos \alpha + \sin \beta))$$

17. Write the following function in the simplest form:

$$\sin^{-1}(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2})$$

18. Solve the following for x:

$$\cos^{-1}((x^2-1)/(x^2+1)) + \tan^{-1}((2x)/(x^2-1)) = 2\pi/3$$

19. Prove that  $\sin^{-1}(63/65) = \sin^{-1}(5/13) + \cos^{-1}(3/5)$ .

20. Find the value of the following:

$$\tan(1/2) \sin^{-1}((2x)/(1+x^2)) + \cos^{-1}((1-y^2)/(1+y^2)), |x| < 1, y > 0$$

and  $xy < 1$ .

21. If  $\cos^{-1}(x/2) + \cos^{-1}(y/3) = 0$ , then prove that

$$9x^2 - 12xy \cos \theta + 4y^2 = 36 \sin^2 \theta.$$

22. Evaluate  $\cot(\sqrt{1+x^2} - x)$

23. Solve the following equation:  $\sin^{-1}(1-x) - 2 \sin^{-1}x = \pi/2$

24. Evaluate  $\tan [2 \tan^{-1}(1/5) + \pi/4]$

25. Solve for x,  $\tan^{-1}(2x) + \tan^{-1}(3x) = \pi/4$

26. Solve for x:  $\tan^{-1}\sqrt{1+x} - \sqrt{1-x} / \sqrt{1+x} + \sqrt{1-x} = \beta$ .

27. Find the solution of the equation  $\tan^{-1}x - \cot^{-1}x = \tan^{-1}(1/\sqrt{3})$ .

# LOYAL EDUCATION MATHEMATICS

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( DAILY PRACTICE PAPER )

[ CLASS XII ]

28. If  $\tan^{-1}(1+1.2) + \tan^{-1}(1/1+2.3) + \dots + \tan^{-1}(1/1+n(n+1)) = \tan^{-1}\theta$ , then find the value of  $\theta$ .

29. Find the principal value of  $\tan^{-1}(\tan(5\pi/6))$

30. Prove that  $\tan^{-1}(\cos x / 1-\sin x) - \cot^{-1}(1+\cos x / 1-\cos x) = \pi/4$ ,  
 $x \in (0, \pi/2)$ .

31. Evaluate  $\tan [1/2 \cos^{-1}(\sqrt{3}/11)]$

32. If  $\tan^{-1}a + \tan^{-1}b + \tan^{-1}c = \pi$ , then prove that  $a + b + c = abc$ .

## ASSESS YOURSELF

1. The equation  $\tan^{-1}x - \cot^{-1}x = \tan^{-1}(1/\sqrt{3})$  has solution as \_\_\_\_\_.

2. If  $\alpha \leq 2 \sin^{-1}x + \cos^{-1}x \leq \beta$ , then  $\alpha =$  \_\_\_\_\_.

3. If  $\tan^{-1}x = \pi/10$  for some  $x \in \mathbb{R}$ , then the value of  $\cot^{-1}x$  is

(a)  $\pi/5$ , (b)  $2\pi/5$ , (c)  $3\pi/5$ , (d)  $4\pi/5$

4. Show that  $\sin^{-1}((a-x)/2a) = 1/2 \cos^{-1}x/a$ .

**Write the principal values in Exercises 5 to 8:**

5.  $\operatorname{cosec}^{-1}(2)$

6.  $\cos^{-1}(-\sqrt{3}/2)$

7.  $\tan^{-1}(-\sqrt{3})$

8.  $\tan^{-1}(\tan(3\pi/4))$

**Write the value in Exercises 9 to 11:**

9.  $\operatorname{cosec}^{-1}(\sqrt{2}) + \sec^{-1}(\sqrt{2})$

10.  $\cos^{-1}(\cos(2\pi/3)) + \sin^{-1}(\cos(2\pi/3))$

11.  $\tan^{-1}(\sqrt{3}) + \cot^{-1}(1/\sqrt{3})$

12. What is the domain of the function  $\operatorname{cosec}^{-1} x$ ?

13. Write one branch of  $\tan^{-1} x$  other than the principal branch.

# LOYAL EDUCATION MATHEMATICS

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[ CLASS XII ]

**Evaluate in Exercises 14 to 20:**

14.  $\sin^{-1} \{ \cos (\sin^{-1}(1/2)) \}$

15.  $\operatorname{cosec}^{-1} \{ \operatorname{cosec} (-\pi/4) \}$

16.  $\cos \{ \pi/3 - \cos^{-1}(1/2) \}$

17.  $\sec^2(\tan^{-1}2)$

18.  $\cos^{-1}(\cos 5\pi/3)$

19.  $\sec^{-1} ((x-3)/(x+3)) + \sin^{-1} ((x+3)/(x-3))$

20.  $\tan^{-1}\{\cos \pi\}$

**Prove that in Exercises 21 to 24:**

21.  $2 \tan^{-1}x = \sin^{-1}(2x)/(1+x^2)$

22.  $2 \cos^{-1}x = \sec^{-1}(1)/(2x^2-1)$

23.  $\sin^{-1}x = \cot^{-1}(\sqrt{(1-x^2)})/x$

24.  $\cos^{-1}x = 2 \cos^{-1}\sqrt{(1+x)}/2$

25. Find the value of  $\operatorname{cosec}(\cot^{-1}y/2)$  in terms of y alone.

**Prove that (Exercises 26 to 34):**

26.  $2 \tan^{-1}(1/5) + \tan^{-1}(1/8) = \tan^{-1}(4/7)$

27.  $\tan^{-1}(1/5) + \tan^{-1}(1/3) + \tan^{-1}(1/7) + \tan^{-1}(1/8) = \pi/4$

28.  $2 \tan^{-1}(1/5) + \tan^{-1}(1/8) = \tan^{-1}(4/7)$

29.  $\cos^{-1}(4/5) + \cos^{-1}(12/13) = \cos^{-1}(33/65)$

30.  $\cos^{-1}(4/5) + \tan^{-1}(3/5) = \tan^{-1}(27/11)$

31.  $\sin^{-1}(1/4) + 2 \tan^{-1}(1/3) = \pi/2$

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[ CLASS XII ]

$$32. \tan^{-1}(3a^2x-x^3)/(a(a^2-3x^2)) = 3 \tan^{-1}(x/a)$$

$$33. \sec^2(\tan^{-1}3) + \operatorname{cosec}^2(\cot^{-1}4) = 27$$

$$34. \sin^{-1}((x + \sqrt{(1-x^2)})/\sqrt{2}) = \pi/4 + \sin^{-1}x, -1 \leq x \leq 1.$$

**Write in the simplest form (Exercises 35 to 38):**

$$35. \cos^{-1}(3/5 \cos x + 4/5 \sin x)$$

$$36. \tan^{-1}(8x)/(1+20x^2)$$

$$37. \cot^{-1}(\sqrt{(1 + \cos 5x)}/(1 - \cos 5x))$$

$$38. \sin^{-1}(x\sqrt{(1-x^2)} + x\sqrt{(1-x^4)})$$

**Solve for x (Exercises 39 to 45):**

$$39. \cos^{-1}x + \sin^{-1}x = \pi/2$$

$$40. \tan(\cos^{-1}x) = \sin(\cot^{-1}(1/2))$$

$$41. \cot^{-1}(2/x) + \cot^{-1}(3/x) = \pi/4$$

$$42. \tan^{-1}((x-3)/(x-4)) + \tan^{-1}((x+3)/(x+4)) = \pi/4$$

[AI 2017]

$$43. 2 \cot^{-1}x + \tan^{-1}x = 2\pi/3$$

$$44. \sin^{-1}(3x/5) + \sin^{-1}(4x/5) = \sin^{-1}x$$

$$45. \tan^{-1}\sqrt{x^2} + x + \sin^{-1}\sqrt{x^2} + x + 1 = \sin^{-1}1$$

$$46. \text{If } \cos^{-1}a + \cos^{-1}b + \cos^{-1}c = \pi, \text{ prove that } a^2 + b^2 + c^2 + 2abc = 1.$$

$$47. \text{If } \tan^{-1}a + \tan^{-1}b + \tan^{-1}c = \pi/2, \text{ prove that } ab + bc + ca = 1.$$

48. Show that

$$2 \tan^{-1}(\tan \alpha/2 \tan \beta/2) = \cos^{-1}((\cos \alpha + \cos \beta)/(1 + \cos \alpha \cos \beta))$$

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[ CLASS XII ]

49. If  $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$ , then prove that  
 $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$ .

50. If  $\sin^{-1}(2a)/(1+a^2) - \cos^{-1}(1-b^2)/(1+b^2) = \tan^{-1}(2x)/(1-x^2)$ , prove  
That  $x = (a-b)/(1+ab)$

51. Find the value of  $\sin\{2 \cot^{-1}(-5/12)\}$ .

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